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remained, until my own researches and my journey to Hungary, the only source of information on these wonderful Magyars. Schmidt succeeded in procuring for Hoüel two copies of Bolyai's work. One Hoüel proceeded to translate himself; the other he sent to Battaglini, asking him to make known in Italy this wonderful idea. This he did by an Italian translation. Thus to Hoüel belongs a perfectly definite and permanent place in the final history of human thought.

Much else he did; so much that I could not attempt to enumerate it in the brief space at my disposal here. Fortunately it has been most sympathetically done by M. G. Brunel in a book of 78 pages most obligingly furnished me by Hoüel's son-in-law, Monsieur H. Barckhausen.

M. Brunel cites on page 34 my Bibliography of Hyper-Space and Non-Euclidean Geometry (1878), and also that published at Kiev in 1880 by Vashtchenko-Zaharchenko, but omits to state that this latter was simply a reprint of mine with slight additions, as is also that given at the end of the Kazan edition of Lobachévski's Works, 1886. Some grotesque effects are produced by reprinting or attempting to reprint my English. Thus under P. G. Tait the title of the work is given as follows: "Mentions Hyper-Space in his Address as Pres. of Math. Sect. of Brit. Assoc. at Edinbvrgh." Under G. P. Young we read "The relation which can be proved to subsist between the Area of a Plane Triangle and the Sum of the Hypothesis that Euclid's twelfth Axiom is false." I must take it, from this extraordinary summation of a hypothesis, that English is nearly as difficult as Russian, though neither can for one instant compete with the Magyar.

In his personal character Hoüel reached that perfection which he has done so much to introduce into the foundations of Geometry.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M. (Princeton); Ph. D. (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from March Number.]

PROPOSITION XXVI. *If the aforesaid AX, BX (fig. 31.) must indeed meet each other, but only at their infinite production toward the parts of the point X: I say there will be no assignable point T in AB, from which a perpendicular erected towards the parts of AX does not at a finite or terminated distance meet this AX in some point F.*

Demonstratur. For (from the preceding hypothesis) there will be in *AX* some point *N*, from which the perpendicular *NK* let fall to *BX* is less than any assigned length, as suppose this *TB*.

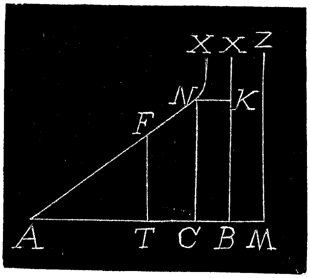

Fig. 31.

But then is assumed in *TB* a portion *CB* equal to *NK*, and *CN* is joined. In the hypothesis of acute angle it is known that the angle *NCB* will be acute. Therefore (from Eu. I. 13) *NCT*, which is the adjacent angle, will be obtuse.

Therefore the straight which is erected toward the parts of *AX* perpendicularly from the point *T* (disposed between the points *A* and *C*), does not meet (from Eu. I. 17) *CN* at any point; and therefore (lest it should enclose a space with *AT*, or with *TC*) it strikes the terminated *AN* in some point *F*.

Therefore even in the hypothesis of acute angle (which we know can here alone hinder) there will be in this *AB* no assignable point *T*, from which the perpendicular erected toward the parts of *AX* does not, at a finite or terminated distance, meet this *AX* in a certain point *F*. Quod etc.

Corollary I. But thence follows, that, point *M* being assumed in *AB* produced, from which towards the parts of the point *X* is erected a perpendicular *MZ*, this cannot, even if infinitely produced, meet the aforesaid *AX*; because otherwise that other straight *BX* must (from the foregoing demonstration) at a finite distance meet this *AX*; which is against the present hypothesis.

Corollary II. From which again follows, that every perpendicular erected from any point, but not however infinitely removed, of this *AB* produced indefinitely, must at a finite distance meet the aforesaid *AX*, as soon as indeed it is assumed that *every* such perpendicular ever more, without any certain limit, approaches the other ever produced straight *AX*.

Corollary III. Whence finally follows, that not even at its infinite production can *BX* be cut by that *AX*; because otherwise from any point of that *AX* beyond the aforesaid intersection a certain perpendicular *ZM* could be supposed let fall to *AB* produced; whence again would follow, that *BX* (against the present hypothesis) met the aforesaid *AX* not at an infinite, but wholly at a finite distance.

But this last dictum is beyond necessity.

[Saccheri here handles a point at infinity, or *figurative* point, as if it were a *proper* point. Upon the extent to which he realized this to be unallowable, depends his real mental attitude toward the non-Euclidean geometries he had discovered. Did he intend his work to suggest what he would not have been allowed to print?]